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Consideration of the Theoretical Possibility of Regulating the Nuclear Reactor by Changing a Fraction of Delayed Neutrons

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Abstract—A lot of theoretical and experimental studies devoted to the effect of external electromagnetic fields and ionization on the beta-decay probability have been published in the past years. The possibility of using this physical effect as the main reactor-regulation mechanism is investigated in this study. A set of equations allowing the operation of a nuclear reactor to be described when the probability for the beta decay of precursors of delayed neutrons and, hence, the fraction of delayed neutrons are functions of time is written and investigated. It is shown that, if the fraction of the delayed neutrons does not change, the proposed set of equations coincides with the generally known one. As follows from the analysis of the solutions to the new set of equations, the proposed reactor-regulation method does not allow reactor runaway driven by prompt neutrons even theoretically. The application of the proposed control method to a circulating-fuel liquid-type reactor is briefly considered.

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1. INTRODUCTION

It has been recognized in the past years that perturbations of atomic electron shells can produce a substantial effect on periods of nuclear decays occurring because of weak and electromagnetic interactions. By way of example, we indicate that the ^{163}Dy, ^{193}Ir, and ^{205}Tl nuclei, which are absolutely stable in a neutral atom, become beta-active when the atom is completely ionized [1] and that a complete ionization of ^{187}Re increased the beta decay probability by a factor of 10^9 (CERN, 1996 [2]). Not only ionization but also a superstrong magnetic field applied to the atom increases beta-decay probabilities [3]. Since the physical mechanism for the production of delayed neutrons (DNs) from predecessor nuclei is directly linked to beta-decay processes, the question of whether it is possible to change the DN fraction was raised in [4]. Later, it was convincingly proven in [5] that the fraction of DNs increases when the atom is ionized or when a superstrong external magnetic field is applied to the atom [6]. In the latter case, the fraction of DNs can increase severalfold.

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The production of DNs in the fission of uranium nuclei was a decisive physical effect that allowed the development of a nuclear reactor, and it underlies operation control of all types of reactors. Delayed neutrons exert a particularly strong effect on the behavior of a circulating-fuel reactor [7]. At the present time, there are no doubts as to whether it is possible to change the DN fraction through an external action. Yet, it is assumed in describing nuclear-reactor kinetics that the fraction of DNs from each particular nuclear emitter does not depend on external conditions [8]. This disagreement is due to the fact that theoretical foundations of reactor operation were developed long before obtaining reliable experimental data indicative of a substantial effect of external physical factors on probabilities of nuclear processes involving weak interactions. At present, only the change in the average DN fraction because of a change in the chemical composition of the core in the course of the reactor run is taken into account in describing reactor kinetics. The objective of the present study is to analyze qualitatively the question of whether there are theoretical grounds for employing a method based on a change in the DN fraction to control a nuclear reactor.

The equations of classical reactor kinetics [7, 8] were basically written for an unchanged DN fraction. Consequently, it would not be correct to analyze these equations in the case of a changing DN fraction. In this study, the equations of reactor kinetics are formulated with allowance for the total number of

DN emitters (including those nuclei whose decay did not result in the production of a neutron). These equations are analyzed in the case of a change in the DN fraction. It is shown that, if the DN fraction is changed through an external action (for example, via the use of a superstrong magnetic field), this may theoretically allow the reactor power to be controlled.

2. KINETICS EQUATIONS WITH ALLOWANCE FOR A POSSIBLE CHANGE IN THE FRACTION OF DELAYED NEUTRONS

The power released in a reactor is proportional to the neutron density n. It is well known [7, 8] that the equations of kinetics within a uniform homogeneous isotropic model can be used to describe qualitatively the DN effect on reactor dynamics. We will use the popular approximation of a single effective DN group for our qualitative analysis of the reactor behavior in response to variations in the beta-decay constants λ. The notation used here is as follows: n is the density of all neutrons in the reactor core; Y is the density of all DN emitters in the core, including those nuclei whose decay did not result in the production of a neutron (this value is considerably different from the emitter density used in classic equations kinetics, where only nuclei whose decay lead to the production of a neutron are taken into account); χ is the prompt-neutron-multiplication coefficient defined as the ratio of the production rate for prompt neutrons to the absorption rate for all neutrons (the ratio of the number of prompt neutrons produced per unit time in a unit volume to the number of all absorbed neutrons in the same volume within the same time); R is the ratio of the number of product DN emitters to the number of product prompt neutrons; T is the effective prompt-neutron-generation lifetime such that, by definition, nT^{-1} is the production rate for prompt neutrons (the number of prompt neutrons produced per unit time in a unit volume); λ_n is the decay constant for DN emitters undergoing beta decay accompanied by neutron production—that is, by definition, $\lambda_n Y$ is the DN production rate (the number of DNs produced per unit time in a unit volume). It is noteworthy that a small number of DN emitters yield more than one DN, and we will take this multiplicity into account in λ_n. Also, λ is the total decay constant for DN emitters with allowance for all their decay channels where an emitter decays to a nucleus that is not a DN emitter (we ignore beta decays that resulted in the production of a new DN emitter, assuming that this nucleus remains one of the nuclei described by the density Y). With the above notation, the equations of balance between the neutrons and the DN emitters have the form

$$\frac{dn}{dt} = \frac{n}{T} - \frac{1}{\chi}\frac{n}{T} + \lambda_n Y, \quad \frac{dY}{dt} = R\frac{n}{T} - \lambda Y. \tag{1}$$

On the right-hand side of the first equation, the first term describes the production of prompt neutrons, the second term describes the absorption of neutrons, and the third term describes DN production. On the right-hand side of the second equation, the first term describes the production of DN emitters and the second term describes their disappearance.

Let us assume that the reactor core is exposed to an action that only slightly affects induced-fission and neutron-absorption processes but increases the beta-decay probability (for example, ionization [5] or a strong magnetic field [6]). In this case, only the coefficients λ_n and λ change in Eqs. (1), while the other quantities remain unchanged. The beta decay of DN-emitter nuclei that lead to the production of a neutron and which proceed to excited levels of intermediate nuclei are known to have appreciably lower decay energies than beta decays not accompanied by the production of a neutron [5, 9]. Consequently, the relative change in λ_n (when ionization occurs or a when one applies a strong external magnetic field) is appreciably larger than the relative change in λ [3–6]. Suppose that the following change in the beta-decay probabilities occurs under the action in question:

$$\lambda = \lambda_0 + \Delta\lambda, \quad \lambda_n = \lambda_{n0} + \Delta\lambda_n.$$

We then have

$$\frac{\Delta\lambda_n}{\lambda_{n0}} > \frac{\Delta\lambda}{\lambda_0}. \tag{2}$$

Let us define the DN fraction η as

$$\eta = R\frac{\lambda_n}{\lambda}. \tag{3}$$

The change of interest in the beta-decay probabilities then leads to an increase in the DN fraction, $\eta = \eta_0 + \Delta\eta$ (η_0 is the unperturbed DN fraction),

$$\frac{\Delta\eta}{\eta_0} = \frac{\lambda_0}{\lambda}\left(\frac{\Delta\lambda_n}{\lambda_{n0}} - \frac{\Delta\lambda}{\lambda_0}\right) > 0. \tag{4}$$

Next, we introduce the reactor reactivity ρ,

$$\rho = \frac{\chi - 1}{\chi} + \eta_0. \tag{5}$$

In terms of the new notation, we find from the set of Eqs. (1) that

$$\frac{dn}{dt} = \frac{\rho - \eta_0}{T}n + \lambda_n Y, \quad \frac{dY}{dt} = R\frac{n}{T} - \lambda Y. \tag{6}$$

Let us consider the behavior of a reactor that, while unperturbed, operated in the steady-state mode—that is, at $\rho = 0$. We are interested in solutions for the initial conditions

$$\frac{Y(0)}{R} = n(0)\frac{\eta_0}{R\lambda_{n0}T} = \frac{n(0)}{\lambda_0 T}. \quad (7)$$

Let us assume that the changes $\Delta\lambda$ and $\Delta\lambda_n$ occur abruptly and instantaneously (within a time much shorter than T) and consider the behavior of the reactor with new time-independent constants $\lambda = \lambda_0 + \Delta\lambda$, $\lambda_n = \lambda_{n0} + \Delta\lambda_n$, and $\rho = 0$. Using (6) and taking into account (4), we obtain the equation

$$\frac{d^2n}{dt^2} + \frac{dn}{dt}\left[\frac{\eta_0}{T} + \lambda\right] - n\frac{\lambda\Delta\eta}{T} = 0, \quad (8)$$

which describes the behavior of the reactor driven from an equilibrium state. If equilibrium was attained under some effect made on the core (at perturbed values of λ and η), the departure from this equilibrium will also take place when the effect is off (perturbation removed). In general, we can therefore consider both positive (the effect is on) and negative (the effect is off) $\Delta\lambda$ and $\Delta\eta$ in (8).

Equation (8) describes an unstable point of the saddle type [10]. It is not difficult to find, via an analysis of eigenvalues of the increment κ, eigensolutions to the equation from the corresponding characteristic equation

$$\kappa = \frac{1}{2}\left(\frac{\eta_0}{T} + \lambda\right)\left[\pm\sqrt{1 + 4\frac{\lambda T\Delta\eta}{(\eta_0 + \lambda T)^2}} - 1\right]. \quad (9)$$

In the approximation of $\lambda T \ll \eta_0$, we obtain

$$\kappa_+ = \lambda\frac{\Delta\eta}{\eta_0}, \quad \kappa_- = -\frac{\eta_0}{T}. \quad (10)$$

The solution in (10) is valid at both small, $\Delta\eta \ll \eta_0$, and large, $\Delta\eta > \eta_0$, perturbations.

When the effect under consideration ($\Delta\eta > 0$) is switched on, the neutron density n will increase with increasing increment κ_+ (10), while, when the effect is switched off, n will stop increasing. Since the power released in the reactor is proportional to the neutron density n [7, 8], we can therefore vary the reactor power by applying external fields to the core. Let us compare this regulation method with the classical one.

3. CLASSIC EQUATIONS OF KINETICS

In order to go over to equations of classic kinetics, we define the quantity C used in [7, 8], which is the density of DN-emitter nuclei that decayed via the neutron-production channel,

$$C = \frac{\lambda_n}{\lambda}Y = \frac{\beta Y}{R}, \quad \beta = \eta. \quad (11)$$

In the classic formulation of the problem, λ_n and λ and, hence, β are constants: only in this case does the substitution of (11) into (6) lead to the known equations [7, 8]

$$\frac{dn}{dt} = \frac{\rho - \beta}{T}n + \lambda C, \quad \frac{dY}{dt} = \frac{\beta n}{T} - \lambda C. \quad (12)$$

It is obvious that, if β is not a constant, the second equation in (12) will be different:

$$\frac{dC}{dt} = \frac{\beta n}{T} - \lambda C + C\frac{d\ln\beta}{dt}.$$

Thus, the set of Eqs. (6) is more general than that in (12) because the latter is only valid at constant λ_n, λ, and β. At a constant reactivity ρ, we find from (12) that

$$\frac{d^2n}{dt^2} + \frac{dn}{dt}\left[\frac{\beta - \rho}{T} + \lambda\right] - n\frac{\lambda\rho}{T} = 0. \quad (13)$$

Analyzing eigenvalues of the increment κ, one can easily find eigensolutions to the above equation from the corresponding characteristic equation

$$\kappa = \frac{1}{2}\left(\frac{\beta - \rho}{T} + \lambda\right) \quad (14)$$

$$\times\left[\pm\sqrt{1 + 4\frac{\lambda T\rho}{(\beta + \lambda T - \rho)^2}} - 1\right].$$

We find that, for $\rho \ll \beta$ and $\lambda T \ll \beta$,

$$\kappa_+ = \frac{\lambda\rho}{\beta}, \quad \kappa_- = -\frac{\beta}{T} \quad (15)$$

and that, for $\rho \sim \beta \gg \lambda T$,

$$\kappa_+ = \frac{\lambda\rho}{\beta - \rho}, \quad \kappa_- = \frac{\rho - \beta}{T}. \quad (16)$$

Equation (13) is similar to Eq. (8) and coincides with it in the first order at small perturbations, $\Delta\eta \ll \eta_0$, if we set

$$\rho = \Delta\eta.$$

However, these equations are qualitatively different at large perturbations, $\Delta\eta \sim \eta_0$. Note that Eqs. (8) and (13) are applicable when $\Delta\eta > \eta_0$ and $\rho > \beta$, respectively, and we know that, in a superstrong magnetic field the DN fraction may become several times larger [6].

In the classic case (13), the signs of the roots $\kappa_\pm$ (16) are reversed if the reactivity becomes larger than the DN fraction, $\rho > \beta$, and it is the large increment $\propto T^{-1}$ that becomes positive—that is, the reactor runaway driven by prompt neutrons will begin, making the reactor uncontrollable. In our case of (8), the inequality $\lambda > 0$ always holds and the sign of the root $\kappa_- \propto T^{-1}$ (9) can never change (10). Consequently, the reactor runaway driven by prompt neutrons (with a large increment $\propto T^{-1}$) will never occur if one employs the proposed new regulation method, and the reactor power will always increase with an increment proportional to λ—that is, in inverse proportion to the lifetime of DN-emitter nuclei.

4. KINETICS OF A CIRCULATING-FUEL REACTOR

Since the problem of kinetics for a circulating-fuel reactor is rather difficult [7], we will consider general qualitative aspects of its model formulation in order to clarify the effect to be produced by a changing DN fraction. We assume that fuel circulates along a closed loop with quite a large length L and a constant cross section. Let the x coordinate be along the direction of fuel motion. In our calculations, the core is assumed to be a regular cylinder and the closure is taken into account via boundary conditions that coincide at $x = 0$ and $x = L$. We assume that the reactor has a small constant negative reactivity $\rho = -\rho_0 < 0$ ($\rho_0 \ll \eta_0$) over its entire length and that the fuel moves at a velocity V. Also, we assume that the core is affected by some action (for example, ionization) over the cross section $x = 0$, with the result that the beta-decay constants for the DN-emitter nuclei and the DN fraction increase in accordance with (4). Since the relative change in λ is appreciably smaller than the relative change in λ_n (2), we ignore the change in λ in this model.

We denote by $\Delta\lambda_{n0}$ the change in the decay constant λ_n for one DN-emitter nucleus, assuming that it occurs discretely when the atom is excited (for example, when it undergoes a complete ionization if we deal with ionization); we also assume that $\Delta\lambda_{n0}$ is identical for all atoms. Let a be the length of the action area such that $a \ll L$, and let I be the fraction of the atoms affected by this action within a unit time. The fraction of affected DN-emitter atoms (the ratio of the number of excited atoms and the total number of atoms) at the exit from the action area in the reference frame comoving with the fuel will then be $aIV^{-1} < 1$. We assume that the number of excited atoms in this reference frame decreases according to an exponential law with a characteristic decrement ν (for example, because of recombination if we deal with ionization). In the problem formulated in this way, the dependence of volume-averaged $\lambda_n(x)$ has the form

$$\lambda_n(x) \equiv \lambda_{n0} + \Delta\lambda_n(x) \tag{17}$$

$$= \lambda_{n0} + \delta\lambda_n \exp\left(-\frac{\nu x}{V}\right), \quad \delta\lambda_n = \Delta\lambda_{n0}\frac{aI}{V},$$

where λ_{n0} is the unexcited value.

To find dominant qualitative effects, we will consider one group of DNs, integrate the kinetic equations [7] over the reactor cross section, and write the resulting equations for the neutron density per meter, n_p, and the DN-emitter density per meter, Y_p. We will assume that the external action does not change radial eigenfunctions for the equation. Under the above assumptions, a procedure similar to that used to derive Eqs. 6 yields

$$\frac{\partial n_p}{\partial t} - V\frac{\partial n_p}{\partial x} = -\frac{\rho_0 + \eta_0}{T}n_p + \lambda_n(x)Y_p, \tag{18}$$

$$\frac{\partial Y_p}{\partial t} - V\frac{\partial Y_p}{\partial x} = R\frac{n_p}{T} - \lambda Y_p.$$

In this set of equations, the variables are separated, and the required solutions are found in the form

$$n_p(x,t) = z(x)\exp(\Omega t), \quad Y_p(x,t) = y(x)\exp(\Omega t).$$

Considering that $\lambda_{n0} = \eta_0\lambda$ and $\lambda T \ll \eta_0$, we obtain

$$\frac{d^2 z}{dx^2} - \frac{dz}{dx}\left(\frac{\rho_0 + \eta_0}{VT} + \frac{\lambda_n'}{\lambda_n} + 2\frac{\Omega}{V}\right) \tag{19}$$

$$+ z\left(\frac{\rho_0\lambda - \Delta\lambda_n(x)}{V^2 T} + F\right) = 0,$$

$$F = \frac{\rho_0 + \eta_0 + \Omega T}{VT}\left(\frac{\lambda_n'}{\lambda_n} + \frac{\Omega}{V}\right) + \frac{\Omega\lambda}{V^2},$$

where the prime designates a derivative with respect to x. By way of example, we consider the case of small perturbations with a long relaxation time, such that $\nu \ll \lambda$; consequently,

$$\frac{\lambda_n'}{\lambda_n} < \frac{\nu}{V} \ll \frac{\lambda T}{VT} \ll \frac{\eta_0}{VT}, \tag{20}$$

$$\eta_0\frac{\lambda_n'}{\lambda_n} < \eta_0\nu\frac{\Delta\lambda_n}{V\lambda_n} \sim \frac{\nu}{\lambda}\frac{\Delta\lambda_n}{V} \ll \frac{\Delta\lambda_n}{V}.$$

In this approximation, we obtain the following equation from (19):

$$V^2 T\frac{d^2 z}{dx^2} - V\eta_0\frac{dz}{dx} \tag{21}$$

$$+ \left[\rho_0\lambda + \eta_0\Omega - \delta\lambda_n \exp\left(-\frac{\nu x}{V}\right)\right] z = 0.$$

This equation has an analytic solution expressed in terms of Bessel functions. The boundary conditions $z(0) = z(L)$ and $z'(0) = z'(L)$ lead to equations for the parameters, and we solve them for Ω.

From the solutions to the equations, we infer that the reactor is capable of operating at the higher-than-critical intensity of the applied action,

$$I_0 = \frac{2}{3}\frac{\rho_0\lambda L\nu}{\Delta\lambda_{n0}a}. \tag{22}$$

Otherwise the chain reaction is not started. If the velocity is as high as the first or second critical velocity,

$$V_{\pm} = \frac{L\nu}{3}\frac{I}{I_0}\left(1 \pm \sqrt{1 - \frac{I_0}{I}}\right), \qquad (23)$$

the reactor operates in the steady-state mode, $\Omega = 0$. If $V_- < V < V_+$, the reactor power increases, whereas, for $V < V_-$ and $V > V_+$ it decreases with the increment

$$\Omega = \frac{\rho\lambda}{\eta_0}\left(\frac{2}{3}\frac{I}{I_0}\frac{L\nu}{V}\left[1 - \frac{L\nu}{6V}\right] - 1\right). \qquad (24)$$

At a small deviation from the first critical velocity, $V = V_- + \Delta V$, the increment of the power change is

$$\Omega = 6\frac{\rho\lambda J(I/I_0)}{\eta_0 L\nu}\Delta V, \qquad (25)$$

where

$$J(u) = u\sqrt{1 - u^{-1}}\left(1 + \sqrt{1 - u^{-1}}\right)^2. \qquad (26)$$

Consequently, the proposed reactor-control scheme allows the reactor power to be regulated by varying the velocity of fuel motion, ΔV, without changing the affecting-action intensity I. Note that the second critical velocity, above which the reactor power decreases, is an additional security safeguard in this regulation method.

5. CONCLUSIONS

Thus, we have seen that, if the DN fraction is varied through the use of an external action (for example, via the application of a superstrong magnetic field), it is theoretically possible to control the reactor power. A reactor is initially subcritical; its startup and subsequent operation is due to the application of an external action to the core. This regulation method will be much safer than the traditional one because, even at large perturbations, the reactor will not experience runaway driven by prompt neutrons and will retain its "controllability."

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REFERENCES

1. M. Jung, F. Bosch, K. Beckert, et al., Phys. Rev. Lett. **69**, 2164 (1992).
2. F. Bosch, T. Faestermann, J. Friese, et al., Phys. Rev. Lett. **77**, 5190 (1996).
3. D. V. Filippov, Yad. Fiz. **70**, 280 (2007) [Phys. At. Nucl. **70**, 258 (2007)]; **70**, 2068 (2007) [Phys. At. Nucl. **70**, 1840 (2007)].
4. D. V. Filippov, A. A. Rukhadze, and L. I. Urutskoev, Ann. Fond. L. de Broglie **29**, Hors Ser. 3, 1207 (2004).
5. A. A. Rukhadze, L. I. Urutskoev, and D. V. Filippov, Yad. Fiz. **69**, 820 (2006) [Phys. At. Nucl. **69**, 792 (2006)]; Yu. P. Gangrsky et al., Yad. Fiz. **71**, 979 (2008) [Phys. At. Nucl. **71**, 951 (2008)].
6. A. A. Rukhadze, L. I. Urutskoev, and D. V. Filippov, Prikl. Fiz., No. 5, 8 (2006).
7. R. Megreblian and D. Holmes, *Reactor Analysis* (McGraw-Hill, New York, 1960; Gosatomizdat, Moscow, 1962).
8. M. Schultz, *Regulation of Nuclear Power Reactors* (Russian translation; Inostr. Liter., Moscow, 1957); N. A. Dollezhal' and I. Ya. Emel'yanov, *The Channel-Type Nuclear Power Reactor* (Atomizdat, Moscow, 1980) [in Russian]; *Theory of Thermal Neutron Nuclear Reactors* (Atomizdat, Moscow, 1957) [in Russian].
9. Yu. P. Gangrsky, B. Dalkhsuren, and B. N. Markov, *Nuclear Fission Fragments* (Énergoatomizdat, Moscow, 1986) [in Russian].
10. L. S. Pontryagin, *Ordinary Differential Equations* (Nauka, Moscow, 1970; Addison-Wesley, Reading, MA, 1962).

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