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NUCLEI
Theory

Decrease in the Probability of Tritium Decay in an External Electric Field

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Abstract—The probability of tritium beta decay is shown to decrease under the effect of a constant uniform external electric field on the atom. For the tritium atom the effect is due first to the reduction of the beta-decay endpoint energy and second to the reduction of the density of vacant bound electron states at the nucleus. Both of these factors reduce the the beta-decay probability: the first reduces the probability decay of to continuum electron states, while the second reduces the probability of decay to a bound state.

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1. INTRODUCTION

At the present time, the helium-isotope mass-spectrometric method [1, 2] makes it possible to determine the beta-decay constant for tritium (atomic or ionized) to a precision of about 0.1%. This precision is sufficient for measuring the difference in the decay constants for the tritium atom and ion and for studying the effect of an external electric field on the beta-decay probability. Changes in the probabilities of allowed and forbidden β^--decays of fully ionized atoms in the field of an intense electromagnetic wave were studied theoretically in [3, 4], where a constant electric field was considered as a particular case. The results reported in [3] show that the total probability λ of beta decay increases in a constant electric field owing to an increase in the decay endpoint energy. The effect is proportional to the square of the electric-field strength (hereafter, we use the system of relativistic units in which $\hbar = c = m_e = 1$, where $\hbar$ is the Planck constant, c is the speed of light in a vacuum, and m_e is the electron mass; the electric-field-strength unit is $E_0 = 1.13 \times 10^{17}$ V/m $= 3.77 \times 10^{12}$ CGS); that is,

$$\frac{\Delta\lambda}{\lambda} = \frac{35}{64}\frac{\alpha E^2}{Q_0^3} \approx 82.8E^2, \qquad (1)$$

where Q_0 is the beta-decay endpoint energy (for tritium, $Q_0 = 18.6$ keV ≈ 0.0364), E is the electric-field strength, and α is the fine-structure constant. In an electric field of strength $E \approx 10^{12}$ V/m $\approx 4 \times 10^7$ CGS $\approx 10^{-5}E_0$, the estimate in (1) yields $\Delta\lambda/\lambda \approx 10^{-8}$.

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In [3, 4], the electric field of the nucleus being considered was disregarded, which entailed the disregard of both beta decay to a bound electron state and the effect of the atomic shell on the beta-decay probability. It is well known that the beta decay of a nucleus in a neutral atom differs from the beta decay of the respective nucleus in the fully ionized atom [1, 2, 5–12]. Here, we will show that, upon takin into account the changes in the atomic shell and decay to a bound state, the result becomes opposite to that in [3]: an external electric field decreases (rather than increases, as in [3]) the probability of tritium beta decay, the effect being six orders of magnitude greater than the estimate in (1).

From [1, 5], it is well known that, in the beta decay of atomic tritium, the probability of decay to a bound electron state is

$$\nu_a \equiv (\frac{\lambda_b}{\lambda})_a = (0.62 \pm 0.07)\%$$

and that the analogous result for the free tritium ion (tritium nucleus deprived of the electron shell) is

$$\nu_t \equiv (\frac{\lambda_b}{\lambda})_t = (1.07 \pm 0.04)\%.$$

We will now find how the density of bound atomic states at the nucleus changes under the effect of an external electric field.

2. CHANGES IN THE DENSITY OF ATOMIC ELECTRONS AT THE NUCLEUS

The problem in the nonrelativistic approximation by means of perturbation theory, retaining terms to the second order (for all functions, $Y = Y^{(0)} + Y^{(1)} +$

$\frac{1}{2}Y^{(2)}$). We consider the equation for an electron in the electric field that is a superposition of the central Coulomb filed of the nucleus and a constant unform external electric field of strength E. It is well known [13–15] that the variables in this equation are separated if used is made of the parabolic coordinates

$$x = \sqrt{\xi\eta}\cos\varphi, \quad y = \sqrt{\xi\eta}\sin\varphi, \quad z = \frac{1}{2}(\xi - \eta), \tag{2}$$
$$r = \sqrt{x^2 + y^2 + z^2} = \frac{1}{2}(\xi + \eta);$$

the volume element is $dV = \frac{1}{4}(\xi + \eta)d\xi d\eta d\varphi$. In the Schrödinger equation

$$\nabla^2\psi + 2\left[W - U(\mathbf{r})\right]\psi = 0 \tag{3}$$

for the electron of energy W in the field specified by the potential energy

$$U(x, y, z) = -\frac{\alpha Z}{r} + E\sqrt{\alpha}z \tag{4}$$
$$= -\frac{2\alpha Z}{\xi + \eta} + \frac{E\sqrt{\alpha}}{2}(\xi - \eta)$$

represent the electron wave function in the form

$$\psi(\xi, \eta, \varphi) = \frac{1}{\sqrt{\pi}}\psi_0 k^{3/2} f(k\xi)g(k\eta)e^{im\varphi}, \tag{5}$$
$$k = \sqrt{-2W},$$

where the functions f and g are normalized as

$$\int_0^\infty f^2(u)du = \int_0^\infty g^2(u)du = 1. \tag{6}$$

From the normalization condition $\int \psi^2 dV = 1$, we obtain

$$\psi_0 = F^{-1/2}, \tag{7}$$
$$F \equiv \frac{1}{2}\int_0^\infty u f^2(ut)du + \frac{1}{2}\int_0^\infty u g^2(u)du.$$

Equation (3) reduces to the set of equation

$$(\hat{H} + \hat{V})f(u) = C_f f(u), \tag{8}$$
$$(\hat{H} - \hat{V})g(u) = C_g g(u)$$

where the constants on the right-hand sides obey the condition

$$C_f + C_g = \alpha Z/k \tag{9}$$

and where

$$\hat{H} \equiv -\frac{d}{du}(u\frac{d}{du}) + \frac{u}{4} + \frac{m^2}{4u}, \tag{10}$$
$$\hat{V} \equiv \frac{E\sqrt{\alpha}}{4k^3}u^2.$$

For a weak external field, we solve the set of Eqs. (8) by the standard methods of perturbation theory [13–15], treating W as a fixed parameter. We construct perturbations for the eigenvalues C in (9), taking V (10) for a small perturbing parameter. The dependence of the energy W and of the wave number k on the external-field strength E is obtained from the constraint in (9). The calculation of the distribution function requires taking into account the fact that perturbation changes not only the wave number k and the functions f and g but also the factor ψ_0, for which expression (7) is exact. One can readily verify that the unperturbed (in the absence of the external field E) solutions to the set of Eqs. (8) have the form [13]:

$$f_{n_1}^{(0)}(u) = I_{n_1+m,n_1}(u), \tag{11}$$
$$g_{n_2}^{(0t)}(u) = I_{n_2+m,n_2}(u),$$
$$C_{n_{1,2}}^{(0t)} = n_{1,2} + \frac{1}{2}(m + 1),$$

where $n_{1,2}$ are the parabolic quantum numbers and

$$I_{n+m,n}(ut) = \sqrt{\frac{n!}{(n+m)!}}e^{-u/2}u^{m/2}L_n^{(m)}(u)$$

are the normalized functions expressed in terms of the Laguerre polynomials $L_n^{(m)}$ [16] (n in the expressions for f and g, we will henceforth imply the parameters n_1 and $n_2, respectively$). For the unperturbed state, the constraint in (9) gives the spectrum

$$C_{n_1}^{(0)} + C_{n_2}^{(0)} = n_1 + n_2 + m + 1 = N, \tag{12}$$
$$k^{(0)} = \alpha Z/N,\ F^{(0t)} = N,$$

where N is the principal quantum number of the unperturbed state.

We expand the exact solutions to the perturbed set of Eqs. (8) in the unperturbed functions $I_{n+m,n}$, which form an orthonormalized basis for various n and fixed m. In order to calculate the change in the probability of allowed beta decay, it is necessary to determine the change in the nonzero density of electrons at the nucleus. Therefore, we will consider $m = 0$ states. The first two perturbation orders yield

$$C_n = (n + \frac{1}{2}) + C_n^{(1)} + \frac{1}{2}C_n^{(2)}, \tag{13}$$
$$C_n^{(1)} = V_{nn},$$
$$C_n^{(2)} = 2\sum_{l\neq n}\frac{V_{nl}^2}{n - l},$$

for f, we have

$$f_n = I_{n,n} + f_n^{(1)} + \tfrac{1}{2}f_n^{(2)}, \tag{14}$$
$$f_n^{t(1)} = \sum_{l\neq n}\frac{V_{nl}}{n - l}I_{l,l},$$

$$f_n^{(2)} = 2\sum_{l\neq n}\left[\sum_{s\neq n}\frac{V_{sn}V_{sl}}{(n-s)(n-l)} - \frac{V_{nn}V_{nl}}{(n-l)^2}\right]I_{l,l} - \sum_{s\neq n}\frac{V_{ns}^2}{(n-s)^2}I_{n,n}.$$

The respective expressions for g are similar. In (13) and (14), we have introduced the matrix elements

$$V_{nn'} \equiv \int f_n^{(0)*}\hat{V}f_{n'}^{(0)}du = \pm U\int u^2 I_{n,n}I_{n',n'}du, \tag{15}$$

where the plus and minus signs refer to f and g, respectively, and

$$U = \frac{1}{4}\frac{E\sqrt{\alpha}}{k^3}. \tag{16}$$

In (15), only the $|n'-n| \leqslant 2$ terms are nonzero. The second order of perturbation theory for the function f in (14) is chosen in such a way as to preserve the normalization in (6). We calculated perturbations, assuming that there is only one perturbation parameter U (16), whereupon we determined the dependence of this parameter on the external-electric-field strength E by using the constraint in (9). In the second order, it is necessary to consider that U depends on k.

With allowing for known values of integrals involving the functions I [13, 16], we find from (13) that

$$C_{n_1}^{(1)} = U\left[6n_1(n_1+1)+2\right], \tag{17}$$
$$C_{n_1}^{(1)} + C_{n_2}^{(1)} = U\cdot 6N(n_1-n_2),$$
$$C_{n_1}^{(2)} = -4U^2\left[34n_1^3 + 51n_1^2 + 35n_1 + 9\right].$$

From the constraint in [9], it can be found that, in the first order, the dependence of the wave number k on U and E has the form

$$k^{(1)}(U) = -\frac{\alpha Z}{N^2}(C_{n_1}^{(1)} + C_{n_2}^{(1)}) = -6U\frac{\alpha Z}{N}(n_1-n_2), \tag{18}$$

$$k^{(1)}(E) = -\frac{3}{2}\frac{E\sqrt{\alpha}}{(\alpha Z)^2}N^2(n_1-n_2).$$

Taking into account relation (6) between U and k, we obtain

$$U(E) = \frac{1}{4}\frac{E\sqrt{\alpha}}{(\alpha Z)^3}N^3 + \frac{9}{8}\frac{E^2}{\alpha^5 Z^6}N^6(n_1-n_2).$$

As a result, we find for k in

$$k^{(2)}(U) = \frac{2\alpha Z}{N^3}(C_{n_1}^{(1)} + C_{n_2}^{(1)})^2 - \frac{\alpha Z}{N^2}(C_{n_1}^{(2)} + C_{n_2}^{(2)}) = 2U^2\frac{\alpha Z}{N} \tag{19}$$
$$\times\,(17N^2 + 87(n_1-n_2)^2 + 19),$$
$$k^{(2)}(E) = 2k^{(1)}(U^{(1)}(E)) + k^{(2)}(U^{(0)}(E))$$
$$= \frac{1}{8}\frac{E^2N^5}{\alpha^4Z^5}(17N^2 - 21(n_1-n_2)^2 + 19).$$

The coefficients of the second terms in (19) are different, as might have been expected for the second order of perturbation theory because of the dependence of U on E and k (16). Taking into account Eq. (5), which relates W to the wave number k, we obtain the well-known spectrum of the Stark effect [13–15]:

$$W^{(0)} = -\frac{1}{2}(k^{(0)})^2 = -\frac{1}{2}(\frac{\alpha Z}{N})^2, \tag{20}$$
$$W^{(1)}(E) = -k^{(0)}k^{(1)} = \frac{3}{2}\frac{E(n_1-n_2)N}{\sqrt{\alpha}Z},$$
$$W^{(2)}(E) = -(k^{(1)})^2 - k^{(0)}k^{(2)}$$
$$= -\frac{1}{8}\frac{E^2N^4}{\alpha^3Z^4}(17N^2 - 3(n_1-n_2)^2 + 19).$$

The ratio of the energy perturbation to the energy of the unperturbed level [13] is a small expansion parameter here:

$$\frac{W^{(1)}}{W^{(0)}} \approx E\sqrt{\alpha}(\frac{N}{\alpha Z})^3 \approx \frac{E\sqrt{\alpha}}{k^3} = 4U \ll 1. \tag{21}$$

The first two orders of perturbation theory (linear and quadratic Stark effects) bad to the splitting of states characterized by a specific value of the principal quantum number N. In the first order of perturbation theory, the total electron density at the nucleus remains unchanged. If $n_1 = n_2$, the energy (and the wave number k) in the first order do not change, the functions f and g are equal in absolute value and are opposite in sign of the perturbations [see Eqs. (14), (15)]. Therefore, the product fg and the sum of the integrals in (7) remain changed, which entails the invariability of ψ at the point $\xi = \eta = 0$ [see Eq. (5)]. If $n_1 \neq n_2$, the pair of states characterized by the parabolic quantum numbers $(n_1,\ n_2)$ and $(n_2,\ n_1)$ have changes in the density that, in the first order, are equal in absolute value and opposite in sign so that the total electron density in the split state characterized by the principal quantum number N remains unchanged in the first order.

In the second order, the changes in the density are nonzero. For the sake of brevity, we will use the notation $I_{n,n} \equiv I_{[n]}$. It follows from (14) that

$$f_n^{(1)}U^{-1} = \frac{1}{2}n(n-1)I_{[n-2]} - 4n^2I_{[n-1]} + 4(n+1)^2I_{[n+1]} - \frac{1}{2}(n+2)(n+1)I_{[n+2]}, \tag{22}$$
$$f_n^{(2)}U^{-2} = I_{[n-4]}\frac{1}{4}n(n-1)(n-2)(n-3)$$

$$
\begin{aligned}
&- I_{[n-3]}\frac{4}{3}n(n-1)(n-2)(3n-2)\\
&+ I_{[n-2]}\cdot 2n(n-1)(8n^2-14n+3)\\
&+ I_{[n-1]}\cdot 4n(n^3+33n^2+3n+3)\\
&- I_{[n]}\frac{1}{2}(65n^4+130n^3+199n^2+134n+34)\\
&+ I_{[n+1]}\cdot 4(n+1)(n^3-30n^2-60n-32)\\
&+ I_{[n+2]}\cdot 2(n+1)(n+2)(8n^2+30n+25)\\
&- I_{[n+3]}\frac{4}{3}(n+1)(n+2)(n+3)(3n+5)\\
&+ I_{[n+4]}\frac{1}{4}(n+1)(n+2)(n+3)(n+4).
\end{aligned}
$$

The first quantum level $1s$ ($N=1$, $n_1=n_2=0$) has the highest density at the nucleus. For this level, we obtain the following changes:

$$
\begin{aligned}
f_0^{(1)} = -g_0^{(1)} = U(4I_{[1]} - I_{[2]}), \qquad (23)\\
f_0^{(2)} = g_0^{(2)}\\
= U^2(-17I_{[0]} - 128I_{[1]} + 100I_{[2]} - 40I_{[3]} + 6I_{[4]}),\\
k^{(1)} = 0, \quad k^{(2)} = 72U^2\alpha Z.
\end{aligned}
$$

The change in the integral in (7) is

$$
\begin{aligned}
F^{(1)} = \int_0^\infty u(f^{(0)}f^{(1)} + g^{(0)}g^{(1)})du = 0, \qquad (24)\\
F^{(2)} = 2U^2\int_0^\infty u(I_{[0]}(-17I_{[0]} - 128I_{[1]})\\
+ (4I_{[1]} - I_{[2]})^2)du = 360U^2.
\end{aligned}
$$

It follows from (5) that the decrease in the density of the electron ground state at the nucleus, ρ_1, under the effect of the external electric field is

$$
\begin{aligned}
\frac{\Delta\rho_1}{\rho_1} = \frac{f^{(2)}}{f^{(0)}} + \frac{g^{(2)}}{g^{(0)}} + \left(\frac{f^{(1)}}{f^{(0)}}\right)^2 \qquad (25)\\
+ 4\frac{f^{(1)}}{f^{(0)}}\frac{g^{(1)}}{g^{(0)}} + \left(\frac{g^{(1)}}{g^{(0)}}\right)^2 + \frac{3}{2}\frac{k^{(2)}}{k^{(0)}} - \frac{1}{2}\frac{F^{(2)}}{F^{(0)}}\\
= -248U^2 = -\frac{31}{2}\frac{E^2}{\alpha^5 Z^6}.
\end{aligned}
$$

Similar calculations for the changes in the total density of the $N=2$, $(n_1, n_2) = (1,0), (0,1)$ excited level yield

$$
\begin{aligned}
f_1^{(1)} = -g_1^{(1)} = U(-4I_{[0]} + 16I_{[2]} - 3I_{[3]}),\\
f_1^{(2)} = g_1^{(2)} = U^2\big(160I_{[0]} - 281I_{[1]} - 968I_{[2]}\\
+ 756I_{[3]} - 256I_{[4]} + 30I_{[5]}\big),\\
k_{0,1}^{(1)} = -k_{1,0}^{(1)} = 3U\alpha Z, \quad k_{0,1}^{(2)} = k_{1,0}^{(2)} = 174U^2\alpha Z,\\
F_{0,1}^{(1)} = -F_{1,0}^{(1)} = 24U, \quad F_{0,1}^{(2)} = F_{1,0}^{(2)} = 2760U^2,\\
\frac{\Delta\rho_2}{\rho_2} = -860\,U^2.
\end{aligned}
$$

Thus, the densities of the ground and excited states of atomic electrons at the nucleus become lower.

3. CHANGES IN THE PROBABILITY OF DECAY TO A BOUND STATE

The probability of decay to a bound state is proportional to the electron density at the nucleus (the density of the state to which decay occurs) and the square of the decay endpoint energy [6]. The change in the endpoint energy for decay to a bound state under the effect of an external electric field is equal to the difference of the changes in the ionization potentials of the final and initial atoms (or ions) [11, 12], that is,

$$
\Delta Q = \left(\left|I_{\mathrm{He}}^{(E)}\right| - \left|I_{\mathrm{He}}^{(0)}\right|\right) - \left(\left|I_{\mathrm{H}}^{(E)}\right| - \left|I_{\mathrm{H}}^{(0)}\right|\right), \quad (26)
$$

where $I^{(E)}$ and $I^{(0)}$ are the ionization potentials of , respectively, the perturbed and unperturbed atoms (or ions) in the external field. In the second order of perturbation theory, the ionization energy increases in absolute value, the increase being in inverse proportion to Z^4 (20). If the tritium atom decays to a bound state, a neutral helium atom featuring two electrons ($Z=2$) appears to be:

$$
\begin{aligned}
\left|I_{\mathrm{He}}^{(E)}\right| - \left|I_{\mathrm{He}}^{(0)}\right| = \frac{9}{2}\frac{E^2}{\alpha^3 Z^4} = \frac{9}{32}\frac{E^2}{\alpha^3}, \qquad (27)\\
\left|I_{\mathrm{H}}^{(E)}\right| - \left|I_{\mathrm{H}}^{(0)}\right| = \frac{9}{4}\frac{E^2}{\alpha^3}.
\end{aligned}
$$

Therefore, the endpoint energy of tritium-atom decay to a bound state decreases

$$
\Delta Q_{ba} = -\frac{63}{32}\frac{E^2}{\alpha^3}. \qquad (28)
$$

In the decay of the tritium ion, the atomic shell is absent in the initial state, so that the ionization energy is zero. Therefore, only the difference of the ionization potentials of the final hydrogen-like helium ion appears in (26). The endpoint energy of tritium-ion beta decay to a bound state increases by

$$
\Delta Q_{bt} = \left|I_{\mathrm{He}^+}^{(E)}\right| - \left|I_{\mathrm{He}^+}^{(0)}\right| = \frac{9}{4}\frac{E^2}{\alpha^3 Z^4} = \frac{9}{64}\frac{E^2}{\alpha^3}. \quad (29)
$$

The decrease in the probability of tritium-atom decay to a bound state is

$$
\begin{aligned}
\left(\frac{\Delta\lambda_b}{\lambda_b}\right)_a = \frac{\Delta\rho}{\rho} + 2\frac{\Delta Q}{Q_0} \qquad (30)\\
= -\frac{E^2}{2\alpha^5}\left(\frac{31}{Z^6} + \frac{63}{8}\frac{\alpha^2}{Q_0}\right)
\end{aligned}
$$

(for the $Z = 2$ final nucleus). The contribution of the second term (which owes its existence to the change in the energy) is about 2.3%. For the tritium ion, we have

$$\left(\frac{\Delta\lambda_b}{\lambda_b}\right)_t = \frac{\Delta\rho}{\rho} + 2\frac{\Delta Q}{Q_0} \tag{31}$$
$$= -\frac{E^2}{2\alpha^5}\left(\frac{31}{Z^6} - \frac{9}{16}\frac{\alpha^2}{Q_0}\right) < 0,$$

the contribution of the second term being $\sim 0.17\%$.

4. CHANGES IN THE PROBABILITY OF DECAY TO THE CONTINUOUS SPECTRUM

The probability of allowed beta decay to electron states belonging to the continuous spectrum is determined by the integral Fermi function [17]. A hydrogen-like helium ion appears to be the final-state system for tritium-atom decay of to of the continuous states spectrum. In this case, one can obtain the change in the decay endpoint energy by means of the same argument as that used to derive (27) and (29). The result is

$$\Delta Q_{ca} = \left(\left|I^{(E)}_{\mathrm{He}^+}\right| - \left|I^{(0)}_{\mathrm{He}^+}\right|\right) - \left(\left|I^{(E)}_{\mathrm{H}}\right| - \left|I^{(0)}_{\mathrm{H}}\right|\right) \tag{32}$$
$$= \left(\frac{9}{64} - \frac{9}{4}\right)\frac{E^2}{\alpha^3} = -\frac{135}{64}\frac{E^2}{\alpha^3}.$$

Since the tritium decay energy is $Q_0 \ll 1$, we can estimate the change in the integral Fermi function in response to the change in the decay endpoint energy by using the leading term of the expansion [17]; that is,

$$S \equiv \int_1^{E_0} E\sqrt{E^2-1}(E_0 - E)^2 dE \tag{33}$$
$$= \frac{1}{60}\sqrt{E_0^2-1}(2E_0^4 - 9E_0^2 - 8)$$
$$+ \frac{1}{4}E_0 \ln\left(E_0 + \sqrt{E_0^2-1}\right) \approx \frac{16\sqrt{2}}{105}Q_0^{7/2},$$

where $E_0 = 1 + Q_0$. For the change in the probability of tritium-atom decay to electron states of the continuous spectrum, we obtain

$$\frac{\Delta\lambda_c}{\lambda_c} = \frac{\Delta S}{S} \approx \frac{8\sqrt{2}}{15}\frac{\Delta Q_{ca}}{Q_0} = -\frac{9\sqrt{2}}{8}\frac{E^2}{\alpha^3 Q_0}. \tag{34}$$

5. TOTAL DECREASE IN THE TRITIUM-DECAY PROBABILITY

The total decrease in the probability λ of tritium-atom beta decay in an external electric field is

$$\left(\frac{\Delta\lambda}{\lambda}\right)_a = \left(\frac{\lambda_b}{\lambda}\right)_a\left(\frac{\Delta\lambda_b}{\lambda_b}\right)_a + \frac{\Delta\lambda_c}{\lambda} \tag{35}$$
$$\approx -\frac{E^2}{2\alpha^5}\left[\nu_a\frac{31}{Z^6} + \frac{9\sqrt{2}}{4}\frac{\alpha^2}{Q_0}\right] \approx -1.85\times 10^8 E^2.$$

In this case, the change in the probability of decay to a bound state in response to the change in the electron density at the nucleus (first term) is on the same order of magnitude as the change in the probability of decay to continuum states in response to the change in the ionization energy (second term).

For the tritium ion, we have

$$\left(\frac{\Delta\lambda}{\lambda}\right)_t = \left(\frac{\lambda_b}{\lambda}\right)_t\left(\frac{\Delta\lambda_b}{\lambda_b}\right)_t \tag{36}$$
$$\approx -\nu_t\frac{E^2}{\alpha^5}\frac{31}{2Z^6} \approx -1.25\times 10^8 E^2,$$

which exceeds in magnitude the estimate in (1) from [3] by a factor of 10^6 and which is opposite to it in sign. The reason for this distinction is the following. The decay of a fully ionized atom only to continuum electron states was considered in [3]. In that case, there is no atomic shell in the initial and in the final state. Therefore, the decay endpoint energy of the cannot change became of the changes in the ionization energy [see Eq. (26)]. For this channel, the beta decay probability may change only because of the increase in the endpoint energy under the effect of an external electric field on the beta electron. This is precisely the mechanism that was considered in [3], but its effect is small (10^{-8}) in relation to the effect of an external electric field on the change in the density of electron bound states at the nucleus of the daughter helium ion [see Eq. (25)]. The analysis in [3] did not include decay to a bound state (λ_b), but this channel always exists, its branching fraction ν_t not being small (1%). It is precisely the changes in λ_b, with allowance for ν_t, that lead to the result presented in (36). For tritium-ion decays, the range of applicability of the estimates obtained here is determined by the condition in (21) at $Z = 2$: $E \ll 3.7\times 10^{-5}E_0 \approx 10^8$ CGS. In the electric field of strength 1.5×10^7 CGS $\approx 4\times 10^{-6}E_0$, the relative decrease in the probability of tritium-ion decay is 0.2%, which is a measurable value [1].

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